Ultracold Gases of Ytterbium: Ferromagnetism and Mott States in an SU(6) Fermi System

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It is argued that ultracold quantum degenerate gas of ytterbium ^{173}Yb atoms having nuclear spin $I = 5/2$ exhibits an enlarged SU(6) symmetry. Within the Landau Fermi liquid theory, stability criteria against Fermi liquid (Pomeranchuk) instabilities in the spin channel are considered. Focusing on the SU($n > 2$) generalizations of ferromagnetism, it is shown within mean-field theory that the transition from the paramagnet to the itinerant ferromagnet is generically first order. On symmetry grounds, general SU(n) itinerant ferromagnetic ground states and their topological excitations are also discussed. These SU($n > 2$) ferromagnets can become stable by increasing the scattering length using optical methods or in an optical lattice. However, in an optical lattice at current experimental temperatures, Mott states with different filling are expected to coexist in the same trap, as obtained from a calculation based on the SU(6) Hubbard model.

I. INTRODUCTION

Recently, the Kyoto group has managed to cool down to quantum degeneracy five Ytterbium isotopes [1]. The Ytterbium atom has a closed-shell electronic structure in the ground state ([Xe] $4f^{14}\ 5s^2\ ^1S_0$), and hence its spin stems *entirely* from the nuclear spin, I. The case of the fermionic species ^{173}Yb is particularly interesting, as it has nuclear spin $I = F = 5/2$. Hence $2F + 1 = n = 6$, and the atom can be in six different internal states. At ultracold temperatures, experiments show that the scattering length is independent of the atom internal state [2]. This can be understood from the absence of electronic spin in the atomic ground state, and the extremely weak dependence of the inter-atomic potential on the atomic nuclear spin. Thus, whereas for a spin-5/2 fermion the Lee-Yang-Huang pseudo-potential, depends on three scattering lengths [3], $a_s^{F=0,2,4}$, the previous observation implies that $a_s^0 = a_s^2 = a_s^4 = a_s$. Mathematically, the interaction part of the Hamiltonian becomes:

$$H_{\rm int} = \frac{4\pi\hbar^2}{M}\sum_{i<j=1}^{N}\left[a_s^0\mathcal{P}_0(ij) + a_s^2\mathcal{P}_2(ij) + a_s^4\mathcal{P}_4(ij)\right]\ \delta(\mathbf{r}_i - \mathbf{r}_j) = \frac{4\pi\hbar^2 a_s}{M}\sum_{i<j=1}^{N}\delta(\mathbf{r}_i - \mathbf{r}_j), \tag{1}$$

where M is the atom mass and $\mathcal{P}_F(ij)$ the projector onto the state of total spin F for the pair of particles i and j. Therefore, the kinetic and interaction terms have the same symmetry, that is, the initial SU(2) spin-symmetry of the Hamiltonian describing an ultracold gas of ^{173}Yb atoms is enlarged to an effective SU(6) symmetry.This is particularly interesting since enlarged symmetries usually lead to additional spectral degeneracies [4], which in turn can lead to exotic (correlated) ground states and topological excitations [5, 6, 7, 8].

The occurrence of SU(6) in an ultracold gas can also lead to new and interesting connections with high-energy physics, where SU(6) has been used to describe the flavor symmetry of spinful quarks, as nuclear forces seem to be spin independent to a first approximation [4]. Indeed, some of the phases discussed below can be regarded as (non-relativistic) pion condensates that spontaneously break SU(6). In addition, these phases also bear some resemblance to the quantum Hall ferromagnets [8] discussed in two-dimensional electron gases with valley symmetry (such as graphene). Their possible existence in ultracold gases of ^{173}Yb can allow for larger control thanks to the large tunability of these atomic systems. In this regard, ultracold ^{173}Yb atoms in optical lattices may also allow the observation of other exotic time-reversal symmetry breaking phases such as the staggered flux phase [9], which has been speculated as the explanation to the anomalous properties of the pseudo-gap phase of the high-T_c cuprate superconductors [10].

In this paper, we study (in Sect. II) the Fermi liquid instabilities in the spin SU($n = 6$) channel of a strongly interacting ^{173}Yb gas. Focusing mainly on ferromagnetism, which breaks the SU(n) symmetry but not the space rotation invariance, we find in Sect. III that the paramagnetic to ferromagnetic transition to be generically first order for $n > 2$ at the mean-field level. On physical and symmetry grounds, we also identify the possible broken-symmetry ground states. The possibility of spontaneously breaking the SU(n) symmetry group in a cascade of phase transitions between different ferromagnetic phases hints at a much

richer phase diagram than in the spin-$\frac{1}{2}$ case [12]. These phases will also sustain exotic topological excitations, such as skyrmions in two dimensions and monopoles in three dimensions. As argued below, these phenomena may be observed by increasing the scattering length using an optical Feshbach resonance [14] or in perhaps also in a deep optical lattice. In Sect. IV, we consider the situation in the lattice. Close to half-filling, *i.e.* 3 atoms per site, many phases, which may [6] or may not [9] break the SU(6) group, are likely to exist. However, at current accessible optical-lattice temperatures, atom hopping is largely incoherent, and Mott states are likely to coexist in the same harmonic trap. Indeed, for an SU(6) Hubbard model at high temperatures, we have computed the density profile showing the Mott plateaux (see Fig. 1). Finally, a summary of the results as well as a brief discussion of how to detect some of the phases discussed here can be found in Sect. V.

II. SU(n) FERMI LIQUID AND FERMI SURFACE INSTABILITIES

We begin our analysis of the ^{173}Yb system, by exploring some consequences of SU($n = 6$) for the Fermi liquid phase of an interacting gas of ^{173}Yb atoms. Although we shall focus on the continuum case, many of there results in this section can be readily applied to the Fermi liquid phase of the gas loaded in an optical lattice (we neglect harmonic confinement for the moment; it will be considered briefly at the end, and more thoroughly elsewhere [16]). Following Landau [15], we describe the low-lying excited states of the system using the distribution function $n^\alpha_\beta(\mathbf{p}) = \langle \psi^\dagger_\beta(\mathbf{p})\psi^\alpha(\mathbf{p})\rangle$ of a set of elementary excitations called Landau quasi-particles (QP, essentially atoms 'dressed' by the interactions). The latter are annihilated (created) by the Fermi operator $\psi^\alpha(\mathbf{p})$ ($\psi^\dagger_\alpha(\mathbf{p})$) carrying (lattice) momentum $\hbar\mathbf{p}$ and SU(n) index $\alpha = 1, \ldots, n$. The excitation free energy of the QP states is given by the Landau functional (summation over repeated Greek-indices is implied henceforth):

$$\delta F = \sum_{\mathbf{p}} \left[\varepsilon^0(\mathbf{p}) - \mu\right] \delta n^\alpha_\alpha(\mathbf{p}) + \frac{1}{2\Omega} \sum_{\mathbf{p},\mathbf{p}'} f^{\alpha\beta}_{\gamma\delta}(\mathbf{p},\mathbf{p}')\delta n^\gamma_\alpha(\mathbf{p})\delta n^\delta_\beta(\mathbf{p}'), \tag{2}$$

where Ω is the system volume, $\epsilon^0(\mathbf{p})$ is the excitation energy of a single Landau quasi-particle carrying momentum $\hbar\mathbf{p}$. We assume the ground state to be an SU(n) singlet and therefore the ground state quasi-particle distribution $[n^0]^\alpha_\beta(\mathbf{p}) = \theta(\mu - \epsilon^0(\mathbf{p}))\,\delta^\alpha_\beta$, where μ is the chemical potential; $\delta n^\alpha_\beta(\mathbf{p}) = n^\alpha_\beta(\mathbf{p}) - [n^0]^\alpha_\beta(\mathbf{p})$. The Landau functions $f^{\alpha\beta}_{\gamma\delta}(\mathbf{p},\mathbf{p}') = f^{\beta\alpha}_{\delta\gamma}(\mathbf{p},\mathbf{p}')$ describe interactions between quasi-particles. The expression for δF can be considerably simplified with the help of group theory by noticing that $\delta n^\alpha_\beta(\mathbf{p})$ transforms as a tensor belonging to the *reducible* representation of SU(n) $n \otimes \bar{n} = 1 \oplus (n^2-1)$, where n and $\bar{n}$ are the fundamental and its complex conjugate representations, whereas 1 is the singlet and n^2-1 the adjoint representations, respectively. Therefore, $\delta n^\alpha_\beta(\mathbf{p}) = \frac{1}{n}\delta\rho(\mathbf{p})\delta^\alpha_\beta + \sum_{a=1}^{n^2-1} \delta m^a(\mathbf{p})\,(\mathbb{T}^a)^\alpha_\beta$, where $\mathbb{T}^a$ are the (traceless) generators of the SU(n) Lie-algebra obeying $\left[\mathbb{T}^a, \mathbb{T}^b\right] = i\sum_{c=1}^{n^2-1} \lambda^{abc}\,\mathbb{T}^c$; choosing the normalization such that $\mathrm{Tr}\left(\mathbb{T}^a\mathbb{T}^b\right) = \frac{1}{2}\delta^{ab}$, the structure constants λ^{abc} are fully anti-symmetric. In this representation, $\delta\rho(\mathbf{p})$ describes the total density fluctuations and $\delta m^a(\mathbf{p})$ the SU(n) magnetization fluctuations. In addition, since the Landau functions transform as tensors belonging to $n\otimes n\otimes \bar{n}\otimes\bar{n} = 1\oplus 1\oplus$ non-singlet representations, all the tensor components are determined by just *two* scalar functions (compared to the five needed for a $F = 5/2$ Fermi gas [3]), that is, $f^{\alpha\beta}_{\gamma\delta}(\mathbf{p},\mathbf{p}') = f^\rho(\mathbf{p},\mathbf{p}')\delta^\alpha_\gamma\delta^\beta_\delta + 2f^m(\mathbf{p},\mathbf{p}')\sum_{a=1}^{n^2-1}(\mathbb{T}^a)^\alpha_\gamma\,(\mathbb{T}^a)^\beta_\delta$.

We next consider the stability of the Fermi surface (FS) of the SU(n) Fermi liquid just described above. For an isotropic FS, general stability conditions against FS deformations and pairing were obtained using the renormalization group by Chitov and Senechal [17]. They concluded that pairing occurs for attractive interactions. For repulsive interactions, d-wave pairing is also possible on a lattice near half-filling, but the pairing temperature rapidly decreases with increasing n [6]. In the case of ^{173}Yb, s-wave interaction between atoms is naturally repulsive, as the scattering length is $a_s = +10.55$ nm [2]. This yields $p_F a_s \simeq 0.1$ at the center of the trap in current experimental conditions [1, 20]. Furthermore, currently accessible temperatures $T/\mu \simeq 0.4$ [1, 20] are well above any pairing temperature scale. Therefore, the Fermi liquid phase should be a good starting description of the system. However, if the interaction is made sufficiently repulsive, the Fermi liquid can become unstable. The stability of the FS to the so-called Pomeranchuk instabilities can be assessed within Fermi liquid theory by considering the excitation energy of a quasi-particle distribution describing a deformation of the FS [15]. In matrix notation: $\mathbb{n}(\mathbf{p}) = \theta\left[\left(\mu - \epsilon^0(\mathbf{p})\right)\mathbb{1} + \delta\mathbb{u}(\hat{\mathbf{p}})\right]$, where $\mathbb{n}(\mathbf{p})$ denotes the matrix whose components are $n^\alpha_\beta(\mathbf{p})$, $\mathbb{1}$ is the unit matrix, and $\delta\mathbb{u}(\hat{\mathbf{p}})$ is a matrix function that describes a small local deformation of the FS ($\hat{\mathbf{p}}$ denotes those $\mathbf{p}$-points lying on the FS). Expanding in powers of $\delta\mathbb{u}(\hat{\mathbf{p}}) = \frac{1}{n}\delta u_\rho(\hat{\mathbf{p}})\mathbb{1} + \sum_a \delta u^a_m(\hat{\mathbf{p}})\mathbb{T}^a$ up to second order, we obtain $\delta\rho(\mathbf{p}) = \delta\left(\mu - \epsilon^0(\mathbf{p})\right)\delta u_\rho(\hat{\mathbf{p}}) + \frac{1}{2!}\delta'\left(\mu - \epsilon^0(\mathbf{p})\right)\left[\frac{1}{n}\left(\delta u_\rho(\hat{\mathbf{p}})\right)^2 + \frac{1}{2}\sum_a\left(\delta u^a_m(\hat{\mathbf{p}})\right)^2\right] + \cdots$ and $\delta m^a(\mathbf{p}) = \delta\left(\mu - \epsilon^0(\mathbf{p})\right)\delta u^a_m(\hat{\mathbf{p}}) + \cdots$ In the continuum or in an optical lattice at low-filling, the FS is isotropic at the locus where $|\mathbf{p}| = p_\mathrm{F}$, where p_F is the Fermi momentum, and $\mathbf{p}/|\mathbf{p}| = \hat{\mathbf{p}}$. For example, in three dimensions, we can expand $\delta u_\rho(\hat{\mathbf{p}}) = \sum_{LM}\delta u^{LM}_\rho\, Y_{LM}(\hat{\mathbf{p}})$, and $\delta u^a_\rho(\hat{\mathbf{p}}) = \sum_{LM}\delta u^{a,LM}_m\, Y_{LM}(\hat{\mathbf{p}})$, $f^{\rho/m}(\mathbf{p},\mathbf{p}') = f^{\rho/m}(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}') = \frac{4\pi}{2L+1}\sum_{LM} f^L_{\rho/m} Y^*_{LM}(\hat{\mathbf{p}})Y_{LM}(\hat{\mathbf{p}}')$, where $Y_{LM}(\hat{\mathbf{p}})$ are the spherical harmonics. Therefore,

$\delta F = \delta F_\rho + \delta F_m$, where (note that $\delta u_\rho^{LM}, \delta u_m^{a,LM}$ have units of energy)

$$\delta F_\rho = \frac{\Omega N^0(\mu)}{8\pi n} \sum_{LM} \left[1 + \frac{\mathcal{F}_L^\rho}{2L+1} \right] \left| \delta u_\rho^{LM} \right|^2 , \tag{3}$$

$$\delta F_m = \frac{\Omega N^0(\mu)}{16\pi} \sum_{LM} \left[1 + \frac{\mathcal{F}_L^m}{2L+1} \right] \sum_a \left| \delta u_m^{a,LM} \right|^2 . \tag{4}$$

We have introduced the (dimensionless) Landau parameters defined as $\mathcal{F}_L^\rho = nN^0(\mu) f_L^\rho$ and $\mathcal{F}_L^m = N^0(\mu) f_L^m$, where $N^0(\mu) = M^* p_F/(2\pi^2\hbar^2)$ is the quasi-particle density of states (per species) at the FS in three dimensions, M^* being the quasi-particle effective mass and $\hbar p_F$ the Fermi momentum ($p_F = (6\pi^2\rho_0/n)^{1/3}$, where ρ_0 is the total density). Hence, from Eqs. (3) and (4), the FS will be unstable if $\mathcal{F}_{\rho/m}^L < -(2L+1)$, for $L = 0, 1, \ldots$

The FS instabilities in the density channel ($\delta F_\rho < 0$, that is, $\mathcal{F}_L^\rho < -(2L+1)$) are formally identical to those occurring in Fermi systems with no spin. They have received much attention recently [18], and lead to phases where the rotation (or point-group, in the lattice) symmetry of the FS is broken ($L > 0$). On the other hand, much less attention has focussed on instabilities in the spin channel, which also break spin symmetry [19], and may occur in the interesting case of the ^{173}Yb system with SU($n = 6$) symmetry. Certainly, the most exotic states will be those resulting from an instability with $L > 0$ in the spin channel, or, for a non-isotropic FS, one that breaks the lattice point-group besides SU(n). The resulting states have a much more complex order parameter, $\Phi_{LM}^a \propto \int d\hat{\mathbf{p}}\, Y_{LM}(\hat{\mathbf{p}})\, (\mathbb{T}^a)_\alpha^\beta \delta n_\beta^\alpha(p_{\rm F}\hat{\mathbf{p}})$, that is the product of an orbital and an SU(n) part (similar to superfluidity in ^{3}He).

However, as ultracold ^{173}Yb atoms naturally interact via *repulsive* s-wave (contact) interactions, in the isotropic case $\mathcal{F}_0^m$ is expected to be the most negative Landau parameter, thus favoring SU(6) ferromagnetic correlations. Indeed, within Hartree-Fock theory, $\mathcal{F}_0^\rho = N_0(\mu) g(n-1) > 0$ whereas $\mathcal{F}_0^m = -N_0(\mu) g < 0$ ($N_0(\mu) = M p_F/(2\pi^2\hbar^2)$, where M is the atom mass, $g = 4\pi\hbar^2 a_s/M$. Hence, the SU(n) generalization of Stoner's criterion for ferromagnetism, $F_0^m < -1$, yields $N_0(\mu) g > 1$, or equivalently, $p_F a_s > p_F a_s^* = \frac{\pi}{2}$, in the continuum case. It is worth noting that this criterion turns out to be the same as for the SU(2) case, that is, it is independent of n. The independence on n of the Stoner criterium in a SU(n) Fermi system can be understood using a simple energetic argument: To create a polarized ground state, imagine for example that $\delta M_0/(n-1)$ fermions are removed from the Fermi surface of each of the $n-1$ flavors with $\alpha < n$ and added to the Fermi surface of the $\alpha = n$ flavor (so that the total particle number is unchanged). For small δM_0, the kinetic energy of the system increases by $(n-1)\frac{[\delta M_0/(n-1)]^2}{2\Omega N_0(\mu)} + \frac{(\delta M_0)^2}{2\Omega N_0(\mu)} = \frac{n(\delta M_0)^2}{2(n-1)\Omega N_0(\mu)}$, whereas the (Hartree-Fock) interaction energy decreases by $\frac{g}{\Omega}\left[\frac{(n-1)(n-2)}{2} \left(\frac{\delta M_0}{n-1} \right)^2 - (n-1) \left(\frac{\delta M_0^2}{n-1} \right) \right] = -\frac{ng(\delta M_0)^2}{2(n-1)\Omega}$. Hence, upon comparing both energies, the dependence on n drops out and the system becomes unstable provided that $N_0(\mu) g > 1$, which is independent of n and agrees with the result obtained from Fermi liquid theory in the Hartree-Fock approximation. The cancellation of the dependence on n to the lowest order is a consequence of the fact that both the kinetic and exchage energies scale linearly with n (inspite of the fact that, naïvely, the interaction scales as n^2). Nevertheless, as we shall see below, the nature of the transition from a paramagnet to an itinerant ferromagnet turns out to be very different for SU(n) with $n > 2$.

III. SU(n) ITINERANT FERROMAGNETS

The previous analysis using Landau Fermi liquid theory does not tell us anything about the order of the transition. In the spin-$\frac{1}{2}$ (SU(2)) case, a Landau free-energy functional obtained from the microscopic Hamiltonian finds a continuous transition [11, 12]. However, it has been recently pointed out that the coupling of the order parameter fluctuations to soft modes changes the order of the transition from second to first order at low temperatures [12, 13]. In order to gain further insights into the nature of the transition at the mean field level, we shall derive in this section an effective action for the ferromagnetic order parameter starting from the microscopic model. To this end, we use the following operator identity for the interaction term of the Hamiltonian density:

$$\mathcal{H}_{\rm int}(\mathbf{r}) = \frac{1}{2} g\, \bar{c}_\alpha(\mathbf{r}) \bar{c}_\beta(\mathbf{r}) c^\beta(\mathbf{r}) c^\alpha(\mathbf{r}) = \frac{(n-1)}{2n} g\, :[\rho(\mathbf{r})]^2: - g \sum_{r=2}^{n} : \left[\bar{c}_\alpha(\mathbf{r})\, (\mathbb{T}^{r^2-1})_\beta^\alpha\, c^\beta(\mathbf{r}) \right]^2 : \tag{5}$$

In the above expression : ... : stands for operator normal order, that is, the prescription that all atom creation fields, $\bar{c}_\alpha(\mathbf{r})$, should stand to the left of the destruction fields, $c^\alpha(\mathbf{r})$. The matrices $\mathbb{T}^{r^2-1} = \frac{1}{\sqrt{2r(r-1)}} \mathrm{diag}\,(1, 1, \ldots, 1-r, \ldots, 0, 0)$ are the diagonal generators of the Lie algebra (*i.e.* the Cartan subalgebra). We next perform a Hubbard-Stratonovich decoupling of the

density ($\propto \rho^2$) and SU(n)-spin interaction terms, which yields the following action ($\beta = 1/T$, T being the absolute temperature)

$$S[\bar{c}_\alpha, c^\alpha, \varphi, \{\phi_r\}] = \int d\mathbf{r} \int_0^{\hbar\beta} \frac{d\tau}{\hbar} \Big\{ \bar{c}_\alpha(\mathbf{r},\tau) \Big[\Big(\hbar\partial_\tau - \mu - \frac{\hbar^2}{2M}\nabla^2 + \frac{n-1}{n}\, g\varphi(\mathbf{r},\tau) \Big) \delta^\alpha_\beta - g \sum_{r=2}^{n} \mathcal{M}_r(\mathbf{r},\tau) (\mathbb{T}^{r^2-1})^\alpha_\beta \Big] c^\beta(\mathbf{r},\tau) - \frac{n-1}{2n} g\varphi^2(\mathbf{r},\tau) + \frac{g}{4} \sum_{r=2}^{n} \mathcal{M}_r^2(\mathbf{r},\tau) \Big\}. \tag{6}$$

Following the work by Hertz [11] for the SU(2) case, we focus on the SU(n) spin fluctuations and therefore obtain an effective action for the fields $\mathcal{M}_r(\mathbf{r},\tau)$ by integrating out the Fermions and setting the density-fluctuation field $\varphi(\mathbf{r},\tau) = \rho_0$ (ρ_0 being the total density), that is, its saddle point value. Such a procedure yields the following effective action:

$$S_{\text{eff}}[\mathbb{M}] = -\mathrm{Tr}\ln\left[-G_0^{-1}\mathbb{1} - g\mathbb{M}\right] + \frac{g}{2\hbar}\int d\mathbf{r} d\tau\, \mathrm{Tr}\,\mathbb{M}^2(\mathbf{r},\tau) \tag{7}$$

where $\mathbb{M}(\mathbf{r},\tau) = \sum_{r=2}^{n} \mathcal{M}_r(\mathbf{r},\tau)\mathbb{T}^{r^2-1}$, such that $\mathrm{Tr}\,\mathbb{M}(\mathbf{r},\tau) = 0$, and $G_0^{-1}(\mathbf{r}-\mathbf{r}',\tau-\tau') = -\left(\hbar\partial_\tau - \mu + \frac{(n-1)}{n} g\rho_0 - \frac{\hbar^2}{2M}\nabla^2\right)\delta(\mathbf{r}-\mathbf{r})\delta(\tau-\tau')$. However, it should be noticed that the present Hubbard-Stratonovich decoupling scheme using only the *diagonal* generators of SU(n) breaks the full SU(n) invariance of the theory. Yet, it does reproduce the correct Stoner criterion in the mean field (Hartree-Fock) approximation, which, as discussed above, comes out to be independent of n. The SU(n) invariance can be recovered by extending the functional integral over the entire set of traceless hermitian matrices $\mathbb{M}$ transforming according to the adjoint representation of SU(n). As noted in Sect. II, a convenient of basis for this set is provided by the generators of the SU(n) Lie algebra, $\mathbb{T}^a$, with $a = 1, \ldots, n^2-1$. Hence, $\mathbb{M}(\mathbf{r},\tau) = \sum_a m_a(\mathbf{r},\tau)\,\mathbb{T}^a$, where $m_a(\mathbf{r},\tau)$ are real fields. Near the paramagnetic-ferromagnetic phase transition, we expect the order parameter to be small and therefore, we perform a series expansion in $\mathbb{M}$ neglecting its dependence in $\mathbf{r}$ and τ. This yields the following (Landau) free-energy per unit volume:

$$\frac{F}{\Omega} = \frac{F_0}{\Omega} + \sum_{n=2} \frac{g^n v_n}{n} \mathrm{Tr}\,\mathbb{M}^n = \frac{F_0}{\Omega} + \frac{g^2}{2} v_2 \mathrm{Tr}\,\mathbb{M}^2 + \frac{g^3}{3} v_3 \mathrm{Tr}\,\mathbb{M}^3 + \frac{g^4}{4} v_4 \mathrm{Tr}\,\mathbb{M}^4 + \cdots \tag{8}$$

The term of $O(g)$ vanishes identically because $\mathbb{M}$ is traceless. However, for $n > 2$, terms of both even and odd order in g are non zero and occur in the free energy expansion in powers of $\mathbb{M}$. This is to be contrasted with the SU($n = 2$) case, where only terms of even order occur [11, 12, 13]. The coefficients $v_2 = (g^{-1} + \chi_2)$ and $v_n = \chi_n$ for $n > 2$, where $\chi_n = \frac{(-1)^n}{\beta\Omega}\sum_k [G_0(k)]^n = -\frac{1}{(n-1)!}\frac{\partial^{n-2} N_0(\mu)}{\partial\mu^{n-2}}$, $k = (i\epsilon_n, \mathbf{k})$ and $G_0(k) = (i\epsilon_n - \epsilon(\mathbf{p}) + \mu)^{-1}$, where $\epsilon_n = \frac{2\pi}{\beta}(n + \frac{1}{2})$, $\epsilon(\mathbf{p}) = \frac{\hbar^2\mathbf{p}^2}{2M}$, and we have shifted the chemical potential $\mu \to \mu - g(n-1)/n\rho_0$ to account for its renormalization due to interactions.

We could have obtained the above free energy based on symmetry considerations of the order parameter. However, the microscopic approach allows us to relate the coefficients of the expansion to the model parameters. We next set $\mathbb{M} = \sum_{a=1}^{n^2-1} m_a \mathbb{T}^a$ in (8) and use the following SU(n) identity (see *e.g.* [22]),

$$\mathbb{T}^a\mathbb{T}^b = \frac{1}{2n}\delta^{ab}\mathbb{1} + \frac{1}{2}\sum_{c=1}^{n^2-1}\left(d^{abc} + if^{abc}\right)\mathbb{T}^c, \tag{9}$$

where the group structure constants d^{abc} are fully symmetric and f^{abc} fully anti-symmetric [22]. For $n = 2$, $d^{abc} = 0$ but for $n > 2$ these structure constants are non-zero, which has important implications for the order of the paramagnetic-ferromagnetic phase transition. In terms of the m_a components of the order parameter, the free-energy reads:

$$\frac{F}{\Omega} = \frac{F_0}{\Omega} + \frac{g^2}{4} v_2 \sum_a (m^a)^2 + \frac{g^3}{12} v_3 \sum_{abc} d^{abc} m_a m_b m_c + \frac{g^4}{16} v_4 \left\{ \frac{1}{n}\left[\sum_a (m_a)^2\right]^2 + \frac{1}{2}\sum_{abcde} d^{abe} d^{cde} m_a m_b m_c m_d \right\} + \cdots \tag{10}$$

The above expression shows explicitly that the Landau free energy contains a cubic term in the order parameter m_a, which implies that, at least at the mean field level, the transition from the paramagnetic to the ferromagnetic phase is first order. Thus, the system will exhibit hysteresis, and phase coexistence, with finite surface tension between the ferromagnetic and paramagnetic phase. Furthermore, the entropy will undergo a finite jump across the phase transition from the paramagnet to the SU(n) ferromagnet. Moreover, the gas parameter resulting from Stoner's criterion $p_F a_s^* = \frac{\pi}{2}$, which is the point where the quadratic coefficient vanishes, is actually *larger* than the critical value of the gas parameter, that is, $p_F a_s^c < p_F a_s^* = \frac{\pi}{2}$. The

latter corresponds to the point where both the paramagnetic minimum ($\mathbb{M}=0$) and ferromagnetic minimum ($\mathbb{M}\neq 0$) have the same free energy.

To illustrate the general ideas presented above, we shall next consider the case of the smaller group SU(3), which already contains essential ingredients of SU($n>2$) ferromagnetism. The more complicated case of SU(6) relevant to an unpolarized mixture of ^{173}Yb atoms will be studied elsewhere [16]. However, it is worth saying that the SU(3) case would correspond to an experiment where the system is prepared as a mixture containing an equal population of *only three* of the six internal states [25] [26] Considering a three dimensional gas in the continuum, setting $\mathbb{M}=U^{\dagger}\left(m_3\mathbb{T}^3+m_8\mathbb{T}^8\right)U$, where $U\in \mathrm{SU}(3)$, and using cyclic property of the trace along with the parametrization $m_3=\frac{(p_Fa_s)^{-1}}{p_F^3}\,\bar{m}_0\,\cos\theta$ and $m_8=\frac{(p_Fa_s)^{-1}}{p_F^3}\bar{m}_0\,\sin\theta$, we arrive at the following expression for the (dimensionless) free-energy at $T\ll\mu$:

$$\frac{(F-F_0)/\mu}{p_F^3\Omega}=\frac{c_2}{2}\left(\frac{1}{p_Fa_s}-\frac{2}{\pi}\right)\,\bar{m}_0^2-\frac{c_3}{3}\bar{m}_0^3\sin 3\theta+\frac{c_4}{4}\bar{m}_0^4-\frac{c_5}{5}\bar{m}_0^5\sin 3\theta+\frac{c_6}{6}\bar{m}_0^6(10-\cos 6\theta)+\ldots$$

In the above expression, the numerical coefficients are $c_2=4\pi, c_3=8\pi/\sqrt{3}, c_4=16\pi^2/3, c_5=320\pi^3/27\sqrt{3}$, and $c_6=16\pi^4/9$. Using the above expression up to sixth order, we can also obtain the shift in the critical gas parameter (relative to the Stoner value, $p_Fa_s^*=\frac{\pi}{2}$): $(p_Fa_s^c)-(p_Fa_s^*)^{-1}\simeq 0.066$ or $1-a_s^c/a_s^*\simeq 0.094\simeq 10\%$ at the the mean field level. Fluctuations are likely to decrease the critical value of the gas parameter even further from the Stoner value, and may also change the character of the transition (see *e.g.* [23]). In the SU($n>2$) case, fluctuations are responsible for the change of the order of the transition for SU(2). Thus, further analysis of the effect of fluctuations is needed but it is beyond the scope of the present work.

For $a_s<a_s^c$ the three energy has one minimum located at $\bar{m}_0=0$. However, for $a_s>a_c$ the free energy exhibits tree degenerate minima, corresponding (in 'cartesian' (m_3,m_8) coordinates) to $\mathbb{M}_0\propto(0,-1)\,\bar{m}_0$ and $\mathbb{M}_0\propto(\pm\sqrt{3}/2,1/2)\,\bar{m}_0$. However, it needs to be noticed that these three minima represent the same physical state, as the result of the invariance of the free energy under the transformation $\theta\to\theta+2\pi j/3$, where $j=1,2$, which corresponds to a cyclic permutation of the SU(3) indices $1\to 2\to 3\to 1$, or in other words, to the existence of three (non-commuting) SU(2) subalgebras in SU(3), into which the larger group $SU(3)$ can be spontaneously broken. Thus, let us choose $\mathbb{M}_0\propto-\bar{m}_0\,\mathbb{T}^8=-\bar{m}_0\,(\mathbb{1}-3\,\mathbf{e}\otimes\mathbf{e}^{\dagger})$ (corresponding to $m_3=0$ and $m_8=-m_0$), where $\mathbf{e}^{\dagger}=(0,0,1)$. This ferromagnetic state corresponds to a gas where one of the species Fermi surface ($\alpha=3$, in this case) grows at the expense of the two others, which remain degenerate. This state left invariant under the transformations generated by the SU(2) subalgebra span by $\{\mathbb{T}^1,\mathbb{T}^2,\mathbb{T}^3\}$. Furthermore, it is left invariant by the U(1) transformations generated by $\mathbb{T}^8$. Thus, the little group of transformations leaving the ground state invariant is $H=\mathrm{SU}(3)\otimes\mathrm{U}(1)$. If the interaction is increased further on, the remaining SU(2) group may be also spontaneously broken down to U(1) in a subsequent transition.

Generally speaking, unlike SU(2) case, the SU(n) may be spontaneously broken in a cascade of phase transitions. A general analysis of the possibilities can be given by considering the structure of the order parameter. As pointed out above, the order parameter is a traceless hermitian matrix, $\mathbb{M}=\sum_a \mathbb{m}_a\mathbb{T}^a$, which transforms according to the adjoint representation of SU(n). Thus, when diagonalized, it has $n-1$ independent eigenvalues. If only $k<n$ of them turn out to be equal, the symmetry breaking pattern (up to discrete groups) will be $\mathrm{SU}(n)\to\mathrm{SU}(k)\times[\mathrm{U}(1)]^{n-k}$. Another more symmetric state occurs when there are only two distinct eigenvalues and hence $\mathrm{SU}(n)\to\mathrm{SU}(n-k)\times\mathrm{U}(k)$ ($k\leq n/2$). When all the $n-1$ eigenvalues turn out to be different, $\mathrm{SU}(n)\to[\mathrm{U}(1)]^{n-1}$, etc. A simple example of the broken symmetry ground states (at the Hartree-Fock level) is provided by the state $|\Phi(p_{\mathrm{F}}^1,\ldots,p_{\mathrm{F}}^n)\rangle=\prod_{\alpha=1}^n\prod_{|\mathbf{p}|<p_{\mathrm{F}}^{\alpha}}c_{\alpha}^{\dagger}(\mathbf{p})|0\rangle$, where $|0\rangle$ is the particle vacuum. The number of different eigenvalues tell us how many of the Fermi momenta coincide. More generally, any FM ground state can be considered to be adiabatically connected with an SU(n) rotation of $|\Phi(p_{\mathrm{F}}^1,\ldots,p_{\mathrm{F}}^n)\rangle$. When there are only two different eigenvalues, the order parameter manifold $\mathcal{M}=G_{n,k}=\mathrm{SU}(n)/[SU(n-k)\times\mathrm{U}(k)]$, that is, a Grassmanian manifold [8]. In particular, for $k=1$, $G_{n,1}\simeq\mathbb{CP}^{n-1}$, the complex projective space. These manifolds have non-trivial second homotopy group, $\pi_2(G_{n,k})=\mathbb{Z}$ ($n\geq 2$), which implies that these FM phases can sustain topologically stable excitations that are skyrmions in $d=2$ and monopoles in $d=3$. Furthermore, when SU(n) breaks into a subgroup containing more than one $U(1)$, $\pi_2(\mathcal{M})=\mathbb{Z}^p$, where $p\leq n-1$ is the total number of $U(1)$'s. The corresponding phases thus support complex types of topological defects described by several (integer) topological charges

Finally, let us mention that as far as the experimental realization of SU($n>2$) ferromagnetism is concerned, the above discussion suggest that the most convenient approach to observe an SU($n>2$) paramagnet to ferromagnet phase transition in the ^{173}Yb system is to increase the scattering length by means of an optical Feshbach resonance [14]. Ferromagnetism may also appear when the system is loaded in an optical lattice. However, this phase will compete with others (see next section) and further analysis will be required to understand the full phase diagram of the lattice system.

IV. ^{173}YB GASES IN AN OPTICAL LATTICE: SU$(n=6)$ HUBBARD MODEL

When the system is loaded in an optical lattice other phases may become more energetically favorable. In a uniformly filled lattice, as the filling $\nu = \mathcal{N}/\mathcal{M}$ ($\mathcal{N}$ and $\mathcal{M}$ being the total atom and site numbers, respectively) approaches half-filling, $\nu = \frac{n}{2}$ other phases can become more favorable than ferromagnetism. Let us assume that the ^{173}Yb loaded in a lattice are accurately described by a single-band Hubbard model, $H = \sum_{\mathbf{p}} \epsilon_0(\mathbf{p})\, c^\dagger_\alpha(\mathbf{p}) c^\alpha(\mathbf{p}) + \frac{U}{2}\sum_{\mathbf{R}} \left[\rho(\mathbf{R})\right]^2$, where $\epsilon_0(\mathbf{p}) = -2t\sum_{i=1}^d \cos k_i a_0$ is the free particle dispersion (a_0 is the lattice parameter), and $\rho(\mathbf{R}) = n^\alpha_\alpha(\mathbf{R})$ the total site occupancy. This model will be accurate when the lattice is sufficiently deep. If the lattice depth is further increased so that hopping is suppressed along one direction, the system dimensionality will become effectively $d = 2$. In this case, there is strong evidence that for large values of n [6, 9], a staggered flux phase [10] with atom currents circulating in opposite directions in neighboring plaquettes will be favored as the ground state. This phase breaks time-reversal as well as lattice translation symmetries but does not break SU(n): thus the system will exhibit long-range order also at finite temperatures in $d = 2$. In Ref. [6] it was argued that $n = 6$ is indeed a borderline case where this phase competes with a flavor density wave that breaks both lattice translation symmetry and SU(n) symmetry. The order parameter of this phase, $D^\alpha_\beta(\mathbf{Q}) = \frac{1}{\Omega}\sum_{\mathbf{p}} \langle c^\dagger_\beta(\mathbf{p}) c^\alpha(\mathbf{p}+\mathbf{Q})\rangle$ (where $\mathbf{Q} = (\pi,\pi)$ at half-filling), is also a tensor belonging to the adjoint representaiton. Thus, if $\mathrm{Tr}\left[\mathbb{D}(\mathbf{Q})\mathbb{T}^a\right] = D^\alpha_\beta(\mathbf{Q})(\mathbb{T}^a)^\beta_\alpha \neq 0$ for any $a = 1,\ldots,n$, the SU(n) symmetry will be broken in one of the the same patterns as in the FM case. Thus, the ^{173}Yb gas in an optical lattice may be an ideal system to study the rich phase diagram resulting from the competition of all these phases.

However, the temperatures that are currently achievable in an optical lattice (typically larger than the hopping amplitude t, see caption on Fig. 1) are well above the temperature scales where the ordered phases discussed above may occur. Furthermore, the presence of the harmonic trap leads to an *inhomogeneous* filling of the lattice. The variation of the site occupation across the trap can be estimated in the so-called atomic (*i.e.* $t = 0$) limit of the SU(n) Hubbard model introduced above upon including the harmonic trap: $H_{\mathrm{at}} = \frac{U}{2}\sum_{\mathbf{R}}\left[\rho(\mathbf{R})\right]^2 + V_{\mathrm{t}}\sum_{\mathbf{R}}\left(\frac{\mathbf{R}}{a_0}\right)^2\rho(\mathbf{R})$, where $V_{\mathbf{t}} = \frac{1}{2}m\omega_0^2 a_0^2$ is the trapping energy and a_0 the (optical) lattice parameter. The average site occupation can be thus obtained from $\langle\rho(\mathbf{R})\rangle = \mathrm{Tr}\left[\rho_{\mathbf{R}} e^{-(H_{\mathrm{at}}-\mu N)/T}\right]/\mathrm{Tr}e^{-(H_{\mathrm{at}}-\mu N)/T}$, with $N = \sum_{\mathbf{R}}\rho(\mathbf{R})$ and T the absolute temperature. Hence,

$$\langle\rho(\mathbf{R})\rangle = T\frac{\partial}{\partial\mu}\ln\left[\sum_{p=0}^{n}\mathcal{C}^n_p e^{-\frac{U}{2T}p^2 - \frac{V_{\mathrm{t}}(\mathbf{R}/a_0)^2-\mu}{T}p}\right], \tag{11}$$

where $\mathcal{C}^n_p = \frac{n!}{p!(n-p)!}$ is the energy degeneracy of a single-site state containing p particles. The chemical potential μ must be adjusted to fix the total number of particles and T must be such that the entropy of the lattice equals that of the gas before *adiabatically* ramping up the lattice. A plot of the site occupancy as a function of the radial distance to the center of the trap $|\mathbf{R}|$ is displayed in Fig. 1.

V. SUMMARY AND CONCLUSIONS

To sum up, by using Fermi liquid theory, we have discussed Fermi liquid (Pomeranchuk) instabilities in the spin chanel of a strongly interacting ultracold ^{173}Yb gas exhibiting an enlarged SU$(n=6)$ symmetry. Focusing on the Ferromagnetic instability, which does not break space rotation or translation symmetries, we have shown that the transition is generically first order (at least, at the mean level). Such an instability corresponds to a phase transition which can observed by increasing the scattering length using an optical Feshbach resonance or/and in an optical lattice. For the continuum case, the first order of the transition implies that the transition takes places at a slightly smaller value of the scattering length than the value provided by the Stoner criterion, which we find to be independent of the order of the group, n. Furthermore, using the smaller group SU(3) as an example, we have illustrated how the larger unitary symmetry is broken by an explicitly analysis of the Landau free energy derived from the microscopic Hamiltonian. Thus, we found that SU(3) is spontaneously broken down to SU$(2)\otimes$ U(1).

On general symmetry grounds, we can expect a number of symmetry-breaking patterns for SU(6), which may be the result of not just one but a cascade of phase transitions between ferromagnetic phases. These SU(n) ferromagnets system can sustain exotic topologically stable excitations, such as skyrmions in $d = 2$ and monopoles in $d = 3$. The resulting phase diagram may be indeed quite rich, and will be explored elsewhere [16]. An interesting direction would be also to apply the analysis, based on Hertz theory [**?**], to study other Fermi surface instabilities in SU(n) spin channel or to the flavor density wave of Ref. [6] on the lattice. Based on the group theoretic properties of the order parameter, the latter may also turn out to be first order at the mean field level. Furthermore, in the optical lattice, the ^{173}Yb system also offers other possibilites. such a realization of the staggered flux phase, which breaks the lattice but not SU(n) symmetry. However, under current experimental conditions, the temperature of the gas in the lattice is well above the ordering temperature for these phases. In this limit, we have obtained the density profile in a harmonic trap (see Fig. 1).

The fact that the spin of the Ytterbium atom in its ground state is entirely nuclear implies that its coupling to a real magnetic field is very weak and this renders magnetic fields impractical to detect the population of different species. However, the ferromagnetic phases and the topological defects discussed above could be detected by means of the optical Stern-Gerlach effect induced by off-resonant circularly polarized light [20]. Although this method has not been yet demonstrated experimentally, it provides the most direct way to image the population of each species in a single shot measurement [20]. Nevertheless, we hope that the possibilities discussed above for the observation of new and exotic many-body states in the ^{173}Yb will spur further theoretical and experimental research along these lines. The first step in this direction may be measuring the site occupation in an optical lattice, which can be carried out as explained in Ref. 21.

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Note added: After completion of our work, we became aware of the work by Gorshkov et al. [24], who also pointed out enlarged SU(n) symmetries of alkaline-earth atomic gases.

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[25] Similarly, smaller unitary groups with $n < 6$ would correspond to mixtures with smaller number of components.
[26] Another possibility is based on the observation that the Lie algebra of SU(6) can be written as a direct product of $\mathrm{SU}(3)_{\mathrm{flavor}} \otimes \mathrm{SU}(2)_{\mathrm{spin}}$, where 'flavor' corresponds to the magnitude of the nuclear spin (that is, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$) and the 'spin' index to the sign ($\pm\frac{1}{2}$, etc.). Thus, we can choose to write the order parameter $\mathbb{M} = \sum'_{fs} m_{fs} \mathbb{u}^f \otimes \mathbb{v}^s$, where $\{\mathbb{u}^f\} = \left\{\mathbb{1}_f, \frac{1}{2}\lambda^1, \ldots, \frac{1}{2}\lambda^8\right\}$ and $\{\mathbb{v}^s\} = \left\{\mathbb{1}_s, \frac{1}{2}\sigma^1, \frac{1}{2}\sigma^2, \frac{1}{2}\sigma^3\right\}$, are the generators of the $SU(3)_{\mathrm{flavor}}$ and $\mathrm{SU}(2)_{\mathrm{spin}}$ Lie algebras, and the prime in the summation means that the operator $\mathbb{1}_f \otimes \mathbb{1}_s$ should be excluded. Assuming that the $\mathrm{SU}(3)_{\mathrm{flavor}}$ can be broken but the $\mathrm{SU}(2)_{\mathrm{spin}}$ cannot, then $\mathbb{M} = \frac{1}{2}\sum_f m_f \lambda^f \otimes \mathbb{1}_s = \mathbb{M}_f \otimes \mathbb{1}_s$. However, it should be notice that this way of breaking the symmetry may not be energetically favorable.

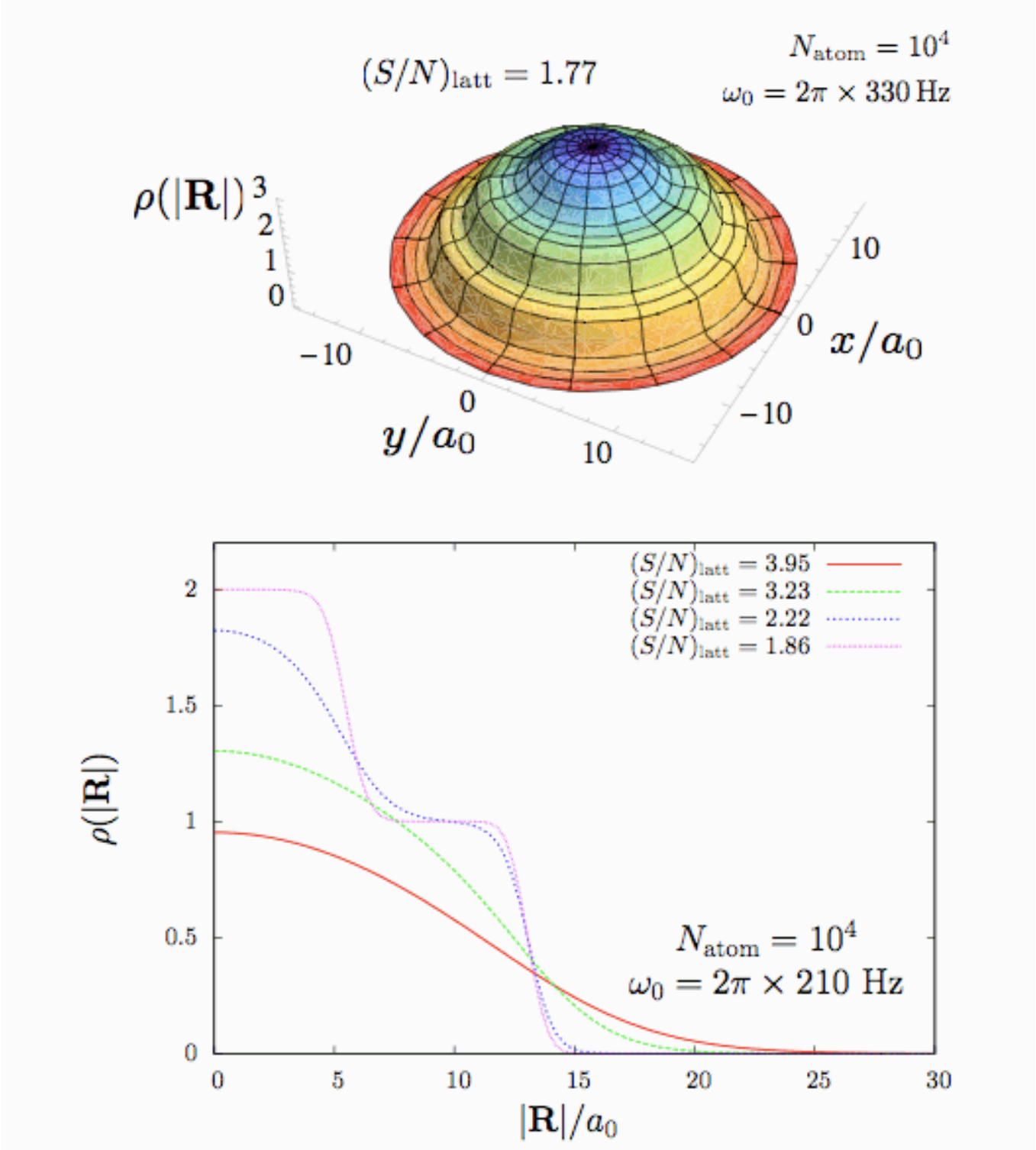

FIG. 1: Site occupation, $\rho(|\mathbf{R}|)$, as a function of the distance to the center of the cloud, $|\mathbf{R}|$, in units of the lattice parameter a_0, obtained from an SU(6) Hubbard model on a cubic lattice in $d = 3$. We have used parameters similar to those of current experiments [1, 20]. In both the upper and lower panel $U/t \simeq 45$ (lattice depth $10\,E_R$). The upper panel shows the emergence of a half-filled ($\rho(|\mathbf{R}|) = 3$) region near the center. The lower panel shows the emergence of the "Mott shell" structure as the lattice entropy per atom is reduced. The red curve corresponds to the currently achievable temperatures of the gas (before *adiabatically* ramping up the lattice) of $T_{\rm init}/T_F = 0.4$ [1, 20] (T_F being the Fermi temperature of the harmonically trapped gas). The entropy is estimated using $(S/N)_{\rm latt} = \pi^2 (T_{\rm init}/T_F)$, for a trapped non-interacting gas. For the smallest value of the entropy, $(S/N)_{\rm latt} = 1.86$, the lattice temperature becomes comparable to the hopping (and therefore the atomic approximation breaks down). Hopping will further reduce the occupation near the edges of the cloud.